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AN EOQ MODEL FOR SUPPLY CHAIN WITH VARIABLE DETERIORATION, SHORTAGES AND PARTIAL BACKLOGGING

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Abstract: In this paper, we have developed a supply chain inventory model for perishable items. In this model, we have considered single manufacturer, multiple distributor and multiple retailer. Demand rate is taken as the exponentially increasing function of time and the production rate is taken as demand dependent which is realistic for newly launched products in the market. Shortages in inventory are allowed and partially backlogged. Cost minimization technique is used to get the expressions for total cost and other parameters. Numerical example is also used to study the behavior of the model and the effects due to change in various parameters have been discussed.

Keywords: Inventory, Supply Chain, Deterioration, Partial backlogging,

1 Introduction

Supply Chain Management (SCM) is one of the most important topics in the study of the management of contemporary manufacturing and distribution. The effective management of supply channel inventories is perhaps the most fundamental objective of SCM. Manufacturers purchase raw material and process them in to finished goods, and sell the finished goods to distributors, then to retailer and/or customer. When an item moves through more than one stage before reaching the final customer, it forms a “multi-echelon” inventory system. A large amount of researches on multi-echelon inventory control has appeared in the literature during the last decades. **Clark (1960)** was the first to study the two-echelon inventory model. They proved the optimality of a base stock policy for the pure serial inventory system and developed an efficient decomposing method to compute the optimal base stock ordering policy. **Banerjee(1986)** considered a joint economic lot size model for purchaser and vendor. **Goyal(1988)** developed a joint economic lot size model for purchaser and vendor. **Lu(1995)** studied one-vendor multi-buyer integrated inventory model.

Grubbstrom(1999) derived an EOQ model with backlogging without use of derivatives. **Hill(1999)** provided the optimal production and shipment policy for the single-vendor single-buyer integrated production inventory problem. **Barron (2001)** algebraically derived the economic production quantity (EPQ) with shortage. **Wee (2002)** obtained the result for economic lot size of the integrated vendor–buyer inventory system. It was derived without using derivatives.**Yang (2004)** discussed an integrated inventory model involving deterministic variable lead time and quality improvement investment. **Sajadieh (2010)** developed an integrated vendor–buyer model with stock-dependent demand.

Wang (2011) considered the case of products with time-sensitive deteriorating rates in a multi-echelon supply chain. **Liao (2013)** formulated a deterministic inventory model for deteriorating items with two warehouses and trade credit in a supply chain system. **Shastri (2013)** suggested a multi-Echelon Supply Chain Management for Deteriorating items with Partial Backordering under Inflationary Environment. **Sarker (2014)** provided an operational policy for a three-stage distributive supply chain system with retailers' backorders. **Barrón (2015)** discussed multi-item EOQ inventory model in a two-layer supply chain while demand varies with promotional effort. **Thomas (2015)** discussed the case of coordination in a multiple producers–distributor supply chain. **Dai (2017)** studied multi-echelon inventory model with three types of demand in supply chain. **Hongfu (2018)** discussed the food supply chain with production disruption and controllable deterioration.

In the present paper supply chain inventory model has been developed for perishable items. In this model, demand rate is taken as the exponential increasing function of time and the production rate is taken as demand dependent which is realistic for newly launched products in the market. Shortages in inventory are allowed and partially backlogged. Cost minimization technique is used to get the expressions for total cost and other parameters. Numerical example is also used to study the behaviour of the model and the effects due to change in various parameters have been considered in the chapter numerical ly.

2 Assumptions and Notations:

The mathematical model of the supply chain inventory problem is based on the following assumptions:

i) Single producer, multi-distributors and retailers are assumed.
ii) A single item is considered which deteriorates with a time dependent rate of deterioration.
iii) Shortages are allowed and partially backlogged.
iv)
v) There is no replacement or repair of deteriorated items.
vi) Time horizon is finite.

In addition, the following notations are used throughout this paper:

$f(t)$ The demand rate at any time t and $f(t) = ae^{bt}$, where a and b are constants. a is the initial demand and b is the rate at which the demand rate itself changes.

$P(t)$ Production rate and $P(t) = \gamma ae^{bt}$, where $\gamma > 1$.

$Q_P(t)$ The inventory level of producer at time t

$Q_D(t)$ The inventory level of distributor at time t

$Q_R(t)$ The inventory level of retailer at time t

T The finite time horizon

$\theta(t)$ The variable deterioration rate such that $\theta(t) = \alpha t$ $(0 < \alpha \ll 1, t \geq 0)$

λ The parameter for partial backlogging, so that that unsatisfied demand is backlogged at a rate $e^{-\lambda t}$, where t is the time up to next replenishment and λ is a positive constant.

N_D Integer number of deliveries from the producer to each distributor during the inventory cycle when there is positive inventory.

N_R Integer number of deliveries from each distributor to his retailer during the inventory cycle when there is positive inventory.

N_{PD} Integer number of distributors supplied by the producer

N_{DR} Integer number of retailers supplied by his distributor

p Maximum inventory level for producer i.e. $Q_P(t_1) = p$

S Maximum shortage level for producer i.e. $Q_P(t_3) = -S$

c_{1P} Holding cost per unit per unit time for producer

c_{1D} Holding cost per unit per unit time for distributor

c_{1R} Holding cost per unit per unit time for retailer

C_{HP} Total holding cost for producer

C_{HD} Total holding cost for distributor

C_{HR} Total holding cost for retailer

c_{2P} Deterioration cost per unit per unit time for producer

c_{2D} Deterioration cost per unit per unit time for distributor

c_{2R} Deterioration cost per unit per unit time for retailer

C_{DP} Total deterioration cost for producer

C_{DD} Total deterioration cost for distributor

C_{DR} Total deterioration cost for retailer

c_{3P} Shortage cost per unit per unit time for producer

c_{3D} Shortage cost per unit per unit time for distributor

c_{3R} Shortage cost per unit per unit time for producer

C_{SP} Total shortage cost for producer

C_{SD} Total shortage cost for distributor

C_{SR} Total shortage cost for retailer

c_{4P} Opportunity cost due to lost sales per unit per unit time for producer

c_{4D} Opportunity cost due to lost sales per unit per unit time for distributor

c_{4R} Opportunity cost due to lost sales per unit per unit time for retailer

C_{OP} Total opportunity cost due to lost sales for producer

C_{OD} Total opportunity cost due to lost sales for distributor

C_{OR} Total opportunity cost due to lost sales for retailer

$c_P^{'}$ Set up cost for producer

$c_D^{'}$ Ordering cost for each distributor for each order

$c_R^{'}$ Ordering cost for each retailer for each order

F Fixed parameter of transportation charge

G Variable parameter of transportation charge

C_{TD} Total transportation cost for each distributor

C_{TR} Total transportation cost for each retailer

R_P Total cost of producer for each cycle

R_D Total cost of distributor each cycle

R_R Total cost of retailer for each cycle

R Total cost for each cycle

K Average cost for each cycle so that $K = \frac{R}{T}$

3 Formulation and solution of the model:

In this paper we have considered one producer which produces the items and delivers in fixed quantities to each distributor during the fixed period. Each distributor in turn delivers the item in fixed quantities to his retailer. To obtain the optimal average cost for the model, we shall discuss three models. These three models are as below:

i) The Producer's Inventory Model
ii) The Distributor's Inventory Model
iii) The Retailer's Inventory Model

We shall discuss these models one by one.

4 The Producer's inventory model:

A single item is considered which deteriorates with time dependent rate of deterioration. The initial inventory of the cycle is zero and production starts at the very beginning of the cycle. As production continues, inventory begins to pile up continuously after meeting demand and deterioration. Production stops at time t_1. The accumulated inventory is just sufficient enough to account for demand and deterioration over the interval $[t_1, t_2]$. Shortage starts after t_2 with the concept of partial backlogging and reach to maximum shortage level at time t_3. Production restarts after t_3 to fulfill the backlog and demand and the cycle ends with zero inventory. The situation is depicted in the following figure 1:

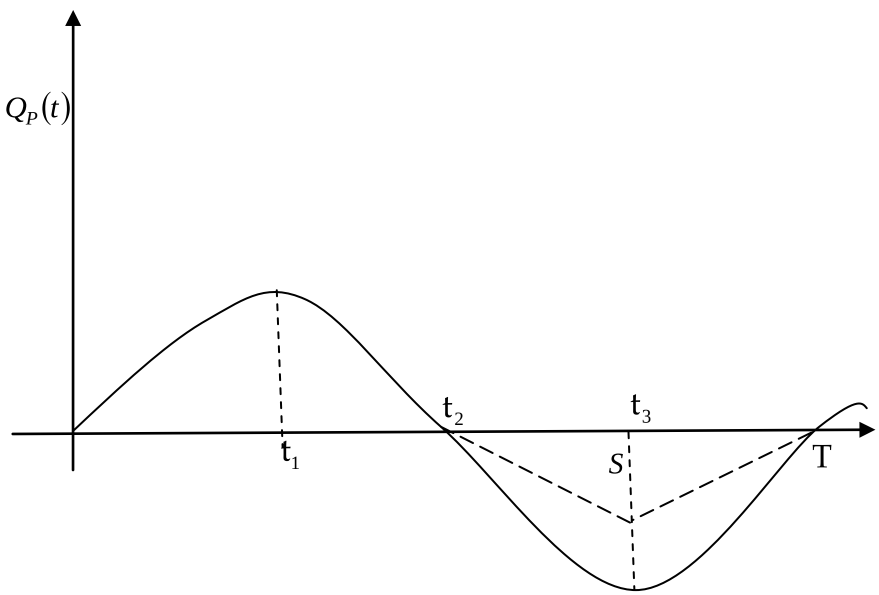

Figure 1: Producer's Model

The differential equations governing the instantaneous states of $Q_P(t)$ in the interval $[0, T]$ are as follows:

$$\frac{dQ_P(t)}{dt} + \theta(t)Q_P(t) = (\gamma - 1)f(t), 0 \leq t \leq t_1 \quad (1)$$

$$\frac{dQ_P(t)}{dt} + \theta(t)Q_P(t) = -f(t), t_1 \leq t \leq t_2 \quad (2)$$

$$\frac{dQ_P(t)}{dt} = -f(t)e^{-\lambda t}, t_2 \le t \le t_3 \qquad (3)$$

$$\frac{dQ_P(t)}{dt} = (\gamma - 1)f(t), t_3 \le t \le T \qquad (4)$$

Conditions are

$Q_P(0) = 0,\ Q_P(t_1) = p, Q_P(t_2) = 0,\ Q_P(t_3) = -S,\ Q_P(T) = 0$

The solutions of equations (1) to (4) are given below:

$$Q_P(t) = \frac{a(-2b^2+2b^2\mathrm{e}^{bt})(-1+\gamma)}{2b^3} + \frac{a(-2+2\mathrm{e}^{bt}-2b\mathrm{e}^{bt}t+b^2t^2)\alpha(-1+\gamma)}{2b^3}, \qquad 0 \le t \le t_1$$

$$Q_P(t) = -\frac{1}{2b^3}a(2b^2\mathrm{e}^{bt} - 2b^2\mathrm{e}^{bt_1} + 2\mathrm{e}^{bt}\alpha - 2\mathrm{e}^{bt_1}\alpha - 2b\mathrm{e}^{bt}t\alpha + b^2\mathrm{e}^{bt_1}t^2\alpha + 2b\mathrm{e}^{bt_1}t_1\alpha - b^2\mathrm{e}^{bt_1}{t_1}^2\alpha) - \frac{1}{2b^3}a\mathrm{e}^{\frac{{t_1}^2\alpha}{2}}(2b^2 - 2b^2\mathrm{e}^{bt_1} + 2\alpha - 2\mathrm{e}^{bt_1}\alpha - b^2t^2\alpha + b^2\mathrm{e}^{bt_1}t^2\alpha + 2b\mathrm{e}^{bt_1}t_1\alpha - b^2{t_1}^2\alpha)(-1+\gamma) t_1 \le t \le t_2$$

$$Q_P(t) = -\frac{a\left[\mathrm{e}^{t(b-\lambda)} - \mathrm{e}^{t_2(b-\lambda)}\right]}{b-\lambda}, t_2 \le t \le t_3$$

$$\mathrm{Q_P}(t) = \frac{a\left(e^{bt} - e^{bT}\right)(-1+\gamma)}{b}, \qquad t_3 \le t \le T$$

The producer's inventory carrying cost during the interval $(0, T)$ is given by

$$C_{HP} = c_{1P}\left[\int_0^{t_1} Q_P(t)dt + \int_{t_1}^{t_2} Q_P(t)dt - N_D N_{PD}\int_0^{\frac{t_2}{N_D}} Q_D(t)dt\right]$$
$$= c_{1P}\Big[-\frac{1}{6b^4}a(-6b^2\mathrm{e}^{bt_1} + 6b^2\mathrm{e}^{bt_2} + 6b^3\mathrm{e}^{bt_1}t_1 - 6b^3\mathrm{e}^{bt_1}t_2 - 12\mathrm{e}^{bt_1}\alpha + 12\mathrm{e}^{bt_2}\alpha + 12b\mathrm{e}^{bt_1}t_1\alpha - 6b^2\mathrm{e}^{bt_1}{t_1}^2\alpha + 2b^3\mathrm{e}^{bt_1}{t_1}^3\alpha - 6b\mathrm{e}^{bt_1}t_2\alpha - 6b\mathrm{e}^{bt_2}t_2\alpha + 6b^2\mathrm{e}^{bt_1}t_1t_2\alpha - 3b^3\mathrm{e}^{bt_1}{t_1}^2t_2\alpha + b^3\mathrm{e}^{bt_1}{t_2}^3\alpha) + \frac{a\mathrm{e}^{bt_1}(b^2+2\alpha-bt_1\alpha)(-1+\gamma)}{b^4} + \frac{1}{6b^4}a(-6b^2 - 6b^3t_1 - 12\alpha - 6bt_1\alpha + b^3{t_1}^3\alpha)(-1+\gamma) + \frac{1}{6b^3}a\mathrm{e}^{\frac{{t_1}^2\alpha}{2}}(t_1 - t_2)(6b^2 - 6b^2\mathrm{e}^{bt_1} + 6\alpha - 6\mathrm{e}^{bt_1}\alpha + 6b\mathrm{e}^{bt_1}t_1\alpha - 4b^2{t_1}^2\alpha + b^2\mathrm{e}^{bt_1}{t_1}^2\alpha - b^2t_1t_2\alpha + b^2\mathrm{e}^{bt_1}t_1t_2\alpha - b^2{t_2}^2\alpha + b^2\mathrm{e}^{bt_1}{t_2}^2\alpha)(-1+\gamma)\Big] - \frac{ac_{1P}(b^2+2\alpha)N_D}{b^4} - \frac{ac_{1P}\mathrm{e}^{\frac{bt_2}{N_D}}(-3b^2N_D^3 - 6\alpha N_D^3 + 3b^3N_D^2t_2 + 6b\alpha N_D^2t_2 - 3b^2\alpha N_Dt_2^2 + b^3\alpha t_2^3)}{3b^4N_D^2} \qquad (5)$$

The producer's deterioration cost during the interval $(0, T)$ is given by

$$C_{DP} = c_{2P}\left[\int_0^{t_1}\theta(t)Q_P(t)dt + \int_{t_1}^{t_2}\theta(t)Q_P(t)dt - N_D N_{PD}\int_0^{\frac{t_2}{N_D}}\theta(t)Q_D(t)dt\right]$$

$$\Rightarrow C_{DP} = \frac{1}{2b} ac_{2P} e^{\frac{t_1{}^2\alpha}{2}}(-1 + e^{bt_1})(-t_1{}^2 + t_2{}^2)\alpha(-1+\gamma) - \frac{1}{2b^3 N_D} ac_{2P}\alpha(2N_D - 2e^{bt_2}N_D - 2N_D{}^2$$

$$+2e^{\frac{bt_2}{N_D}}N_D{}^2 - b^2N_D t_1{}^2 + b^2e^{bt_1}N_D t_1{}^2 + 2be^{bt_2}N_D t_2 - 2be^{\frac{bt_2}{N_D}}N_D t_2 + b^2e^{\frac{bt_2}{N_D}}t_2{}^2 - b^2e^{bt_1}N_D t_2{}^2$$

$$-2N_D\gamma + 2e^{bt_1}N_D\gamma - 2be^{bt_1}N_D t_1\gamma + b^2N_D t_1{}^2\gamma) \qquad (6)$$

The producer's shortage cost during the interval $(0, T)$ is given by

$$C_{SP} = c_{3P}\left[\int_{t_2}^{T} -Q_P(t)dt\right]$$

$$\Longrightarrow C_{SP} = -c_{3P}\left[-\frac{ae^{bt_3}(-1+\gamma)}{b^2} + \frac{ae^{bT}(1-bT+bt_3)(-1+\gamma)}{b^2} - \frac{ae^{t_3(b-\lambda)}}{(b-\lambda)^2} + \frac{ae^{t_2(b-\lambda)}(1-bt_2+bt_3+t_2\lambda-t_3\lambda)}{(b-\lambda)^2}\right]$$

(7)

The producer's opportunity cost due to lost sales during the interval $(0, T)$ is given by

$$C_{OP} = c_{4P}\left[\int_{t_2}^{t_3}(1-e^{-\lambda t})f(t)dt\right]$$

$$\Longrightarrow C_{OP} = c_{4P}\left[\frac{a(-e^{bt_2}+e^{bt_3})}{b} + \frac{a\{e^{t_2(b-\lambda)}-e^{t_3(b-\lambda)}\}}{b-\lambda}\right] \qquad (8)$$

Total cost of producerR_P= $c'_P + C_{HP} + C_{DP} + C_{SP} + C_{OP}$, where C_{HP}, C_{DP}, C_{SP} and C_{OP} are given by equations (5) to (8).

5 The Distributor's inventory model:

The distributor's inventory model is depicted in the figure 2:

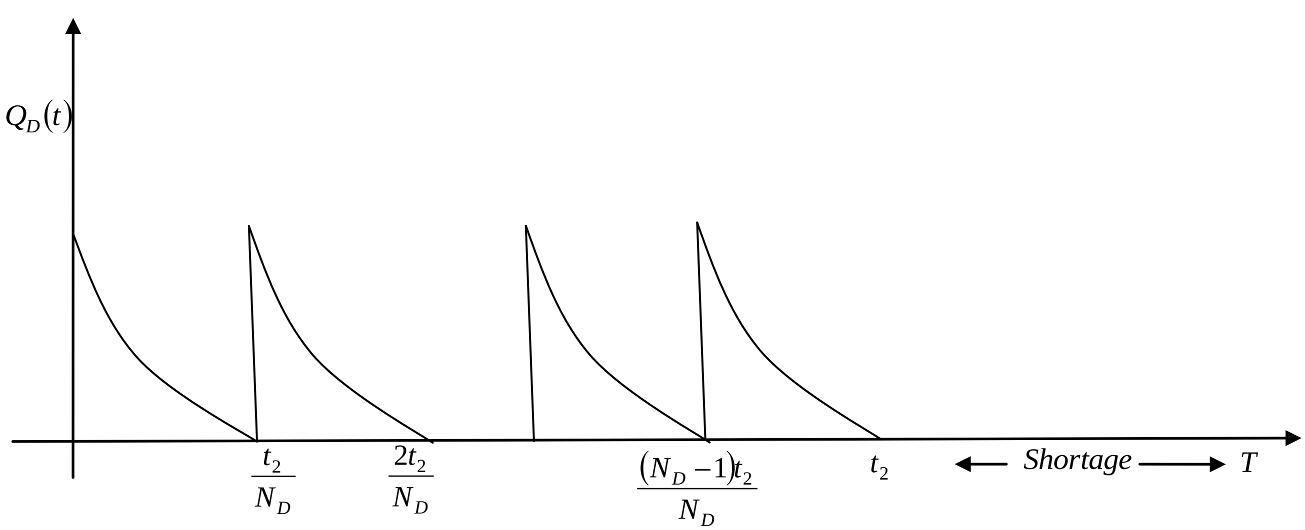

Figure 2: Distributor's Model

For a distributor, there are N_D deliveries during the inventory cycle. When there is positive inventory, the distributor inventory level is governed by the following differential equation:

$$\frac{dQ_D(t)}{dt} + \theta(t)Q_D(t) = -\frac{f(t)}{N_{PD}}, 0 \le t \le \frac{t_2}{N_D} \qquad (9)$$

The solution to above equation is given by,

$$\mathrm{Q_D(t)} = -\frac{ae^{bt}(b^2+\alpha-bt\alpha)}{b^3N_{\mathrm{PD}}} - \frac{ae^{\frac{bt_2}{N_D}}(-2b^2{N_D}^2-2{N_D}^2\alpha+b^2{N_D}^2t^2\alpha+2bN_Dt_2\alpha-b^2{t_2}^2\alpha)}{2b^3{N_D}^2N_{\mathrm{PD}}}$$

The transportation charge per shipment is assumed to be linear function of lot size. It is given by following relation:

Total transportation cost for distributor $C_{TD} = [F + GQ_D(0)]N_D$

$$C_{TD} = \frac{N_D(-ab^2G+b^3FN_{\mathrm{PD}}-aG\alpha)}{b^3N_{\mathrm{PD}}} + \frac{a\mathrm{e}^{\frac{bt_2}{N_D}}G(2b^2{N_D}^2+2{N_D}^2\alpha-2bN_Dt_2\alpha+b^2{t_2}^2\alpha)}{2b^3N_DN_{\mathrm{PD}}} \qquad (10)$$

The distributor's inventory carrying cost during the interval $(0, T)$ is given by

$$C_{HD} = c_{1D}\left[N_D\int_0^{\frac{t_2}{N_D}} Q_D(t)dt - N_RN_{DR}\int_0^{\frac{t_2}{N_R}} Q_R(t)dt\right]$$

$$\Rightarrow C_{HD} = -\frac{ac_{1D}\mathrm{e}^{\frac{bt_2}{N_R}}(-3b^2{N_R}^3+3b^3{N_R}^2t_2-6{N_R}^3\alpha+6b{N_R}^2t_2\alpha-3b^2N_R{t_2}^2\alpha+b^3{t_2}^3\alpha)}{3b^4N_{\mathrm{PD}}{N_R}^2}$$
$$-\frac{1}{3b^4{N_D}^2N_{\mathrm{PD}}}ac_{1D}(-3b^2{N_D}^3 + 3b^2e^{\frac{bt_2}{N_D}}{N_D}^3 + 3b^2{N_D}^2N_R - 3b^3e^{\frac{bt_2}{N_D}}{N_D}^2t_2 - 6{N_D}^3\alpha +$$
$$6e^{\frac{bt_2}{N_D}}{N_D}^3\alpha + 6{N_D}^2N_R\alpha - 6be^{\frac{bt_2}{N_D}}{N_D}^2t_2\alpha + 3b^2e^{\frac{bt_2}{N_D}}N_D{t_2}^2\alpha - b^3e^{\frac{bt_2}{N_D}}{t_2}^3\alpha) \qquad (11)$$

The distributor's deterioration cost during the interval $(0, T)$ is given by

$$C_{DD} = c_{2D}\left[N_D\int_0^{\frac{t_2}{N_D}} \theta(t)Q_D(t)dt - N_RN_{DR}\int_0^{\frac{t_2}{N_R}} \theta(t)Q_R(t)dt\right]$$

$$\Rightarrow C_{DD} = -\frac{ac_{2D}e^{\frac{bt_2}{N_R}}(2{N_R}^2-2bN_Rt_2+b^2{t_2}^2)\alpha}{2b^3N_{\mathrm{PD}}N_R}$$
$$+\frac{ac_{2D}\left(-2{N_D}^2+2e^{\frac{bt_2}{N_D}}{N_D}^2+2N_DN_R-2be^{\frac{bt_2}{N_D}}N_Dt_2+b^2e^{\frac{bt_2}{N_D}}{t_2}^2\right)\alpha}{2b^3N_DN_{\mathrm{PD}}} \qquad (12)$$

The distributor's shortage cost during the interval $(0, T)$ is given by

$$C_{SP} = c_{3D}\left[\int_{t_2}^{T} -Q_P(t)dt\right]$$

$$\Rightarrow C_{SP} = -c_{3D}\left[-\frac{ae^{bt_3}(-1+\gamma)}{b^2} + \frac{ae^{bT}(1-bT+bt_3)(-1+\gamma)}{b^2} - \frac{ae^{t_3(b-\lambda)}}{(b-\lambda)^2} + \frac{ae^{t_2(b-\lambda)}(1-bt_2+bt_3+t_2\lambda-t_3\lambda)}{(b-\lambda)^2}\right] \quad (13)$$

The distributor's opportunity cost due to lost sales during the interval $(0, T)$ is given by

$$C_{OD} = c_{4D}\left[\int_{t_2}^{t_3}(1-e^{-\lambda t})f(t)dt\right]$$

$$\Rightarrow C_{OD} = c_{4D}\left[\frac{a(-e^{bt_2}+e^{bt_3})}{b} + \frac{a\{e^{t_2(b-\lambda)}-e^{t_3(b-\lambda)}\}}{b-\lambda}\right] \quad (14)$$

Total cost of distributors $R_D = (N_D c'_D + C_{TD} + C_{HD} + C_{DD} + C_{SD} + C_{OD})N_{PD}$, where C_{TD}, C_{HD}, C_{DD}, C_{SD} and C_{OD} are given by equations (10) to (14).

6 The Retailer's inventory model:

The retailer's inventory model is depicted in the figure 3:

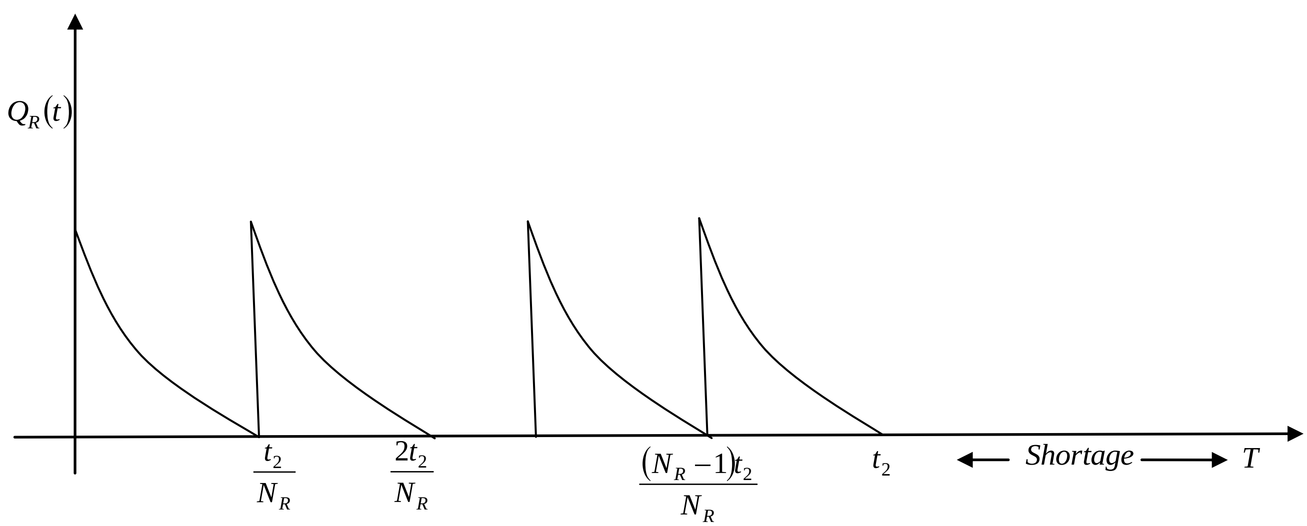

Figure 3: Retailer's Model

For a retailer, there are N_R deliveries during the inventory cycle. When there is positive inventory, the retailer's inventory level is governed by the following differential equation:

$$\frac{dQ_R(t)}{dt} + \theta(t)Q_R(t) = -\frac{f(t)}{N_{PD}N_{DR}}, 0 \le t \le \frac{t_2}{N_R} \quad (15)$$

The solution to above equation is given by,

$$Q_R(t) = -\frac{ae^{bt}(b^2+\alpha-bt\alpha)}{b^3 N_{\mathrm{DR}} N_{\mathrm{PD}}} - \frac{ae^{\frac{bt_2}{N_R}}(-2b^2{N_R}^2-2{N_R}^2\alpha+b^2{N_R}^2t^2\alpha+2bN_Rt_2\alpha-b^2{t_2}^2\alpha)}{2b^3 N_{\mathrm{DR}} N_{\mathrm{PD}} {N_R}^2}$$

Total transportation cost for retailer $C_{TR} = [F + GQ_R(0)]N_R$

$$\mathrm{C_{TR}} = \frac{N_R(-ab^2G+b^3FN_{\mathrm{DR}}N_{\mathrm{PD}}-aG\alpha)}{b^3 N_{\mathrm{DR}} N_{\mathrm{PD}}} + \frac{ae^{\frac{bt_2}{N_R}}G(2b^2{N_R}^2+2{N_R}^2\alpha-2bN_Rt_2\alpha+b^2{t_2}^2\alpha)}{2b^3 N_{\mathrm{DR}} N_{\mathrm{PD}} N_R} \quad (16)$$

The retailer's inventory carrying cost during the interval $(0,T)$ is given by

$$C_{HR} = c_{1R}N_R\int_0^{\frac{t_2}{N_D}} Q_R(t)dt$$

$$\Rightarrow C_{HR} = -\frac{ac_{1R}e^{\frac{bt_2}{N_R}}(3b^2{N_R}^3-3b^3{N_R}^2t_2+6{N_R}^3\alpha-6b{N_R}^2t_2\alpha+3b^2N_R{t_2}^2\alpha-b^3{t_2}^3\alpha)}{3b^4 N_{\mathrm{DR}} N_{\mathrm{PD}} {N_R}^2} + \frac{ac_{1R}N_R(b^2+2\alpha)}{b^4 N_{\mathrm{DR}} N_{\mathrm{PD}}} \quad (17)$$

The retailer's deterioration cost during the interval $(0,T)$ is given by

$$C_{DR} = c_{2R}N_R\int_0^{\frac{t_2}{N_R}} \theta(t)Q_R(t)dt$$

$$\Rightarrow C_{DR} = -\frac{ac_{2R}N_R\alpha}{b^3 N_{\mathrm{DR}} N_{\mathrm{PD}}} + \frac{ac_{2R}e^{\frac{bt_2}{N_R}}(2{N_R}^2-2bN_Rt_2+b^2{t_2}^2)\alpha}{2b^3 N_{\mathrm{DR}} N_{\mathrm{PD}} N_R} \quad (18)$$

The retailer's shortage cost during the interval $(0,T)$ is given by

$$C_{SR} = c_{3R}\left[\int_{t_2}^{T} -Q_P(t)dt\right]$$

$$\Longrightarrow C_{SR} = -c_{3R}\left[-\frac{ae^{bt_3}(-1+\gamma)}{b^2} + \frac{ae^{bT}(1-bT+bt_3)(-1+\gamma)}{b^2} - \frac{ae^{t_3(b-\lambda)}}{(b-\lambda)^2} + \frac{ae^{t_2(b-\lambda)}(1-bt_2+bt_3+t_2\lambda-t_3\lambda)}{(b-\lambda)^2}\right]$$
(19)

The retailer's opportunity cost due to lost sales during the interval $(0,T)$ is given by

$$C_{OR} = c_{4R}\left[\int_{t_2}^{t_3}(1-e^{-\lambda t})f(t)dt\right]$$

$$\Longrightarrow C_{OD} = c_{4R}\left[\frac{a(-e^{bt_2}+e^{bt_3})}{b} + \frac{a\{e^{t_2(b-\lambda)}-e^{t_3(b-\lambda)}\}}{b-\lambda}\right] \quad (20)$$

Total cost of retailers $R_R = (N_Rc'_R + C_{TR} + C_{HR} + C_{DR} + C_{SR} + C_{OR})N_{DR}$, where C_{TR} , C_{HR}, C_{DR}, C_{SR} and C_{OR} are given by equations (16) to (20).

Now, average cost K of model is given by,

$$K = \frac{1}{T}(R_P + R_D + R_R),$$

where R_P, R_D and R_R are obtained in above three models.

The optimum values of t_1,t_2, $t_3$and T which minimize average cost K are obtained by using the equations:

$$\frac{\partial K}{\partial t_1} = 0\,, \frac{\partial K}{\partial t_2} = 0, \frac{\partial K}{\partial t_3} = 0 \quad and \quad \frac{\partial K}{\partial T} = 0$$

7 Numerical Example

To illustrate the model numerically, we use the following parameter values:

$c_{1P} = 2, c_{1D} = 3, c_{1R} = 4, c_{2P} = 3, c_{2D} = 5, c_{2R} = 7, c_{3P} = 1, c_{3D} = 3.5, c_{3R} = 4$
$c_{4P} = 6, c_{4D} = 7, c_{4R} = 8,\ c'_P = 1000, c'_D = 500, c'_R = 200,\ a = 1,\ b = 0.2, \alpha = 0.01,$
$\lambda = 0.1,\ \gamma = 2,\ N_D = 10,\ N_R = 20, N_{PD} = 30, N_{DR} = 40\ ,\ F = 1,\ G = 1$

Applying the subroutine FindRoot in Mathematica 8, we obtain the optimal solution for t_1, t_2, $t_3$and T as follows:

$t_1 = 3.25128, t_2 = 4.60506,\ t_3 = 5.57700,\ T = 6{,}21522$

Also, the optimal average cost for these parameters is 50375.4

8 Sensitivity Analysis

Sensitivity analysis is performed by changing (increasing and decreasing) the parameters by 10%, 30% and 50%, and taking one parameter at a time, keeping the remaining parameters at their original values. Thus following table is formed:

Table1

Changing Parameter	**% Change**	t_1	t_2	t_3	T	**Average Cost**	**% change in average cost**
c_{1P}	+50	3.28361	4.66826	5.60162	6.22725	49418.3	-1.91
	+30	3.28052	4.63932	5.58690	6.22274	49619.8	-1.50
	+10	3.26507	4.62223	5.58251	6.21113	49791.1	-1.22
	−10	3.25922	4.57584	5.51655	6.14292	50960.0	1.16
	−30	3.25001	4.54842	5.49796	6.12700	51091.8	1.42
	−50	3.24390	4.52071	5.47757	6.10890	51243.6	1.72
c_{1D}	+50	3.25750	4.61112	5.58123	6.21890	50345.0	-0.06
	+30	3.25432	4.60800	5.57925	6.21725	50359.3	-0.03
	+10	3.25113	4.60492	5.57712	6.21542	50373.6	0.00
	−10	3.24971	4.60353	5.57590	6.21432	50382.6	0.01
	−30	3.24847	4.60229	5.57481	6.21321	50391.5	0.03
	−50	3.24826	4.60207	5.57457	6.21271	50395.2	0.04

c_{1R}	+50	3.25121	4.60501	5.57700	6.21528	50375.0	0.00
	+30	3.25121	4.60504	5.57701	6.21527	50375.2	0.00
	+10	3.25122	4.60511	5.57701	6.21527	50375.4	0.00
	−10	3.25130	4.60513	5.57693	6.21525	50375.5	0.00
	−30	3.25132	4.60514	5.57692	6.21524	50375.7	0.00
	−50	3.25133	4.60515	5.57685	6.21519	50375.8	0.00
c_{2P}	+50	3.31171	4.61125	5.57826	6.15110	50892.1	1.03
	+30	3.29050	4.60791	5.57675	6.15251	50642.4	0.53
	+10	3.26362	4.60666	5.55653	6.15422	50435.9	0.12
	−10	3.23263	4.58755	5.52557	6.18583	50330.1	-0.09
	−30	3.20565	4.58364	5.52542	6.21235	50078.2	-0.59
	−50	3.17826	4.58001	5.52531	6.21625	49806.2	-1.13
c_{2D}	+50	3.25128	4.60506	5.57712	6.21522	50367.1	-0.02
	+30	3.25128	4.60506	5.57712	6.21522	50375.4	0.00
	+10	3.25128	4.60506	5.57712	6.21522	50375.4	0.00
	−10	3.25128	4.60507	5.57712	6.21522	50375.4	0.00
	−30	3.25129	4.60507	5.57712	6.21522	50375.4	0.00
	−50	3.25129	4.60507	5.57712	6.21522	50375.4	0.00
c_{2R}	+50	3.25128	4.60506	5.57712	6.21522	50375.4	0.00
	+30	3.25128	4.60506	5.57712	6.21522	50375.4	0.00
	+10	3.25128	4.60506	5.57712	6.21522	50375.4	0.00
	−10	3.25128	4.60506	5.57712	6.21522	50375.4	0.00
	−30	3.25128	4.60506	5.57712	6.21522	50375.4	0.00
	−50	3.25128	4.60506	5.57712	6.21522	50375.4	0.00
c_{3P}	+50	3.25771	4.61153	5.58195	6.21877	49972.4	-0.80
	+30	3.25768	4.61132	5.58095	6.21782	50354.2	-0.04
	+10	3.25455	4.60825	5.57893	6.21652	50364.8	-0.02
	−10	3.24837	4.60222	5.57526	6.21406	50385.0	0.02
	−30	3.24777	4.60162	5.57468	6.21382	50386.9	0.02
	−50	3.23888	4.58273	5.52166	6.14839	50914.6	1.07
c_{3D}	+50	3.94554	5.26515	5.98436	6.48683	48242.4	-4.23
	+30	3.67258	5.00982	5.81091	6.35425	49255.5	-2.22
	+10	3.39579	4.7395	5.63664	6.31189	50220.9	-0.31
	−10	3.10243	4.45417	5.48803	6.23312	50776.2	0.80
	−30	2.71277	3.95917	5.42319	6.16690	50967.1	1.17
	−50	2.63594	3.31840	5.14635	6.11855	51223.5	1.68
c_{3R}	+50	4.27408	5.58584	6.24426	7.10314	43026.8	-14.59
	+30	3.88802	5.20960	6.09987	6.70848	45998.3	-8.69
	+10	3.47313	4.81291	5.94192	6.45182	47995.1	-4.73
	−10	3.05706	4.44341	5.67955	6.25894	52504.2	4.23
	−30	2.68256	4.04074	5.55392	5.96192	55010.3	9.20
	−50	2.43912	3.68456	4.80128	5.65874	57047.9	13.25
c_{4P}	+50	3.22375	4.56822	5.55356	6.14403	50452.3	0.15
	+30	3.23244	4.57655	5.56822	6.14650	50431.1	0.11
	+10	3.24769	4.60154	5.57461	6.21375	50387.6	0.02
	−10	3.25901	4.61261	5.58171	6.21838	50349.4	-0.05
	−30	3.26823	4.62116	5.58500	6.21878	50345.1	-0.06
	−50	3.27235	4.62514	5.58766	6.21998	50334.9	-0.08
c_{4D}	+50	2.70365	3.51983	5.48777	8.23619	48284.3	-4.15
	+30	2.94056	4.05754	5.64396	7.79600	48936.8	-2.86

	+10	3.10348	4.45505	5.78439	6.61704	49623.9	-1.49
	−10	3.40507	4.74885	5.86416	6.39896	51095.3	1.43
	−30	3.74402	5.08011	6.13345	6.23947	51895.3	3.02
	−50	4.13552	5.45153	6.22099	6.16520	52796.7	4.81
c_{4R}	+50	2.24967	3.00934	5.22440	8.19254	44790.8	-11.09
	+30	2.69415	3.54581	5.69105	7.78424	46940.8	-6.82
	+10	3.06885	4.45326	5.96941	7.27282	48906.2	-2.92
	−10	3.49075	4.82989	6.06454	6.84556	51439.3	2.11
	−30	4.04847	5.36628	6.08721	6.45794	53895.3	6.99
	−50	4.70856	6.02046	6.61985	6.16795	56130.7	11.42
c'_P	+50	3.25128	4.60506	5.57701	6.21524	50455.7	0.16
	+30	3.25128	4.60506	5.57701	6.21523	50423.6	0.10
	+10	3.25128	4.60506	5.57700	6.21522	50391.5	0.03
	−10	3.25129	4.60507	5.57700	6.21522	50359.4	-0.03
	−30	3.25129	4.60507	5.57699	6.21521	50327.3	-0.10
	−50	3.25129	4.60507	5.57699	6.21520	50295.2	-0.16
c'_D	+50	3.25396	4.60813	5.58033	6.21900	62404.9	23.88
	+30	3.25358	4.60801	5.57980	6.21813	57588.8	14.32
	+10	3.25157	4.60515	5.57735	6.21581	52784.0	4.78
	−10	3.25102	4.60498	5.57661	6.21457	47966.9	-4.78
	−30	3.25055	4.60372	5.57479	6.21242	43154.4	-14.33
	−50	3.25004	4.60267	5.57292	6.21001	38339.9	-23.89
c'_R	+50	3.25392	4.60812	5.58041	6.21914	63207.5	25.47
	+30	3.25352	4.60800	5.57986	6.21823	58070.4	15.28
	+10	3.25159	4.60516	5.57738	6.21585	52944.5	5.10
	−10	3.25101	4.60498	5.57658	6.21453	47806.3	-5.10
	−30	3.25063	4.60375	5.57469	6.21225	42672.6	-15.29
	−50	3.25003	4.60256	5.57261	6.20958	37537.3	-25.48
a	+50	3.24999	4.60257	5.57129	6.20752	50516.7	0.28
	+30	3.25063	4.60269	5.57307	6.21024	50463.3	0.17
	+10	3.25185	4.60524	5.57623	6.21394	50401.6	0.05
	−10	3.25297	4.60783	5.57960	6.21780	50338.6	-0.07
	−30	3.25411	4.60818	5.58115	6.22037	50286.1	-0.18
	−50	3.25861	4.61404	5.58636	6.22554	50212.4	-0.32
b	+50	2.15271	2.99077	4.27309	5.07058	61833.2	22.74
	+30	2.55877	3.61919	4.65945	5.35642	57470.1	14.08
	+10	2.95356	4.28492	5.44522	5.95218	52851.3	4.91
	−10	4.65278	5.39853	6.01258	6.19832	48565.9	-3.59
	−30	5.01329	5.91085	6.52892	7.55545	44017.1	-12.62
	−50	5.56944	6.21345	6.98675	8.34936	39921.8	-20.75
α	+50	3.19778	4.58642	5.52551	6.15117	50884.4	1.01
	+30	3.22249	4.58769	5.53532	6.16208	50647.6	0.54
	+10	3.24232	4.60526	5.54705	6.17535	50464.2	0.18
	−10	3.26215	4.60836	5.55803	6.18652	50191.6	-0.36
	−30	3.27413	4.60994	5.56835	6.19445	50097.2	-0.55
	−50	3.27983	4.61121	5.57894	6.20415	49983.5	-0.78
λ	+50	1.82783	2.61527	3.45218	5.40296	61305.0	21.70
	+30	2.40386	3.20013	4.67913	5.62200	57143.0	13.43
	+10	3.01950	3.79427	5.40922	6.13779	53572.8	6.35
	−10	3.59337	4.95984	5.71648	6.25768	49995.3	-0.75

	−30	4.20981	5.55753	6.48543	6.90064	45311.1	-10.05
	−50	4.67067	6.21234	7.79549	7.08620	41659.8	-17.30
γ	+50	4.55524	5.84988	6.20248	6.69352	45897.0	-8.89
	+30	4.09995	5.40684	5.94122	6.46439	47151.4	-6.40
	+10	3.55189	4.89470	5.70760	6.29068	49539.2	-1.66
	−10	3.02924	4.39825	5.42153	6.15957	51005.1	1.25
	−30	2.59359	4.09806	5.07730	5.93023	52360.2	3.94
	−50	2.16282	3.50697	4.70692	5.70257	54017.5	7.23
N_D	+50	3.24731	4.60190	5.57624	6.21481	62461.7	23.99
	+30	3.24810	4.60242	5.57638	6.21528	57629.7	14.40
	+10	3.25112	4.60506	5.57674	6.21530	52790.4	4.79
	−10	3.25146	4.60506	5.57725	6.21561	47960.3	-4.79
	−30	3.25519	4.60827	5.57802	6.21576	43120.0	-14.40
	−50	3.25870	4.61138	5.580240	6.21705	38272.8	-24.02
N_R	+50	3.25657	4.61099	5.58263	6.22118	63251.1	25.56
	+30	3.25431	4.60819	5.58014	6.21874	58104.4	15.34
	+10	3.25369	4.60792	5.57932	6.21735	52944.5	5.10
	−10	3.25001	4.60362	5.57554	6.21363	47800.4	-5.11
	−30	3.24906	4.60219	5.57313	6.21057	42645.2	-15.35
	−50	3.24122	4.58359	5.51913	6.14317	37869.3	-24.83
N_{PD}	+50	3.22397	4.56855	5.51320	6.14045	63248.9	25.56
	+30	3.23153	4.57575	5.51741	6.14388	58308.9	15.75
	+10	3.24850	4.60232	5.57472	6.21293	52819.2	4.85
	−10	3.25603	4.60991	5.57873	6.21956	47917.8	-4.88
	−30	3.27238	4.62498	5.58132	6.22314	43046.6	-14.55
	−50	3.28102	4.62970	5.58766	6.23442	38316.6	-23.94
N_{DR}	+50	3.21635	4.56100	5.50832	6.13609	64165.1	27.37
	+30	3.22623	4.57052	5.51406	6.14089	58859.6	16.84
	+10	3.23873	4.58255	5.52126	6.14688	53552.0	6.31
	−10	3.26344	4.61661	5.58298	6.21863	47745.3	-5.22
	−30	3.27915	4.63215	5.59044	6.21951	42478.1	-15.68
	−50	3.30455	4.65287	5.59487	6.23092	37368.9	-25.82
F	+50	3.25127	4.60506	5.57701	6.21524	50463.8	0.18
	+30	3.25128	4.60506	5.57701	6.21523	50428.4	0.11
	+10	3.25128	4.60506	5.57700	6.21522	50393.1	0.04
	−10	3.25129	4.60507	5.57700	6.21522	50357.8	-0.03
	−30	3.25129	4.60507	5.57699	6.21521	50322.5	-0.11
	−50	3.25129	4.60507	5.57699	6.21520	50287.1	-0.18
G	+50	3.24779	4.60170	5.57529	6.21415	50384.9	0.02
	+30	3.24781	4.60171	5.57539	6.21430	50383.9	0.02
	+10	3.24956	4.60340	5.57613	6.21465	50380.4	0.01
	−10	3.25471	4.60838	5.58026	6.21630	50366.2	-0.02
	−30	3.25477	4.60843	5.57871	6.21644	50365.0	-0.02
	−50	3.25603	4.60985	5.57879	6.21789	50352.9	-0.04

From Table 1, the following points are noted:

i) It is seen that the percentage change in the optimal cost is almost equal for both positive and negative changes of all the parameters.

ii) The optimal cost increases (decreases) with the increase (decrease) in the value of the parameters c_{2P}, c'_P, c'_D, c'_R,c_{4P},a, b, α, λ,N_D, N_R, N_{PD}, N_{DR}, F and G.This trend is reversed for the parameters c_{1P}, c_{1D},c_{3P}, c_{3D}, c_{3R}, c_{4D}, c_{4R} andγ.

iii) Model is highly sensitive to changes inc_{3R} , c_{4R},c'_D, c'_R, b, λ,N_D, N_R, N_{PD} andN_{DR}.Model is moderately sensitive to changes in γ . It has low sensitivity to c_{1P}, c_{1D},c_{2P},c_{3P}, c_{3D}, c_{4D}, c'_P,a, α, F and G.Model is almost insensitive to changes in c_{1R},c_{2R} and c_{2D}.

iv) From the above points, it is clear that much care is to be taken to estimatec_{3R} , c_{4R},c'_D, c'_R, b, λ,N_D, N_R, N_{PD} and N_{DR}.

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